Bilateral Trade with Interdependent Values

CDS Seminar Talk

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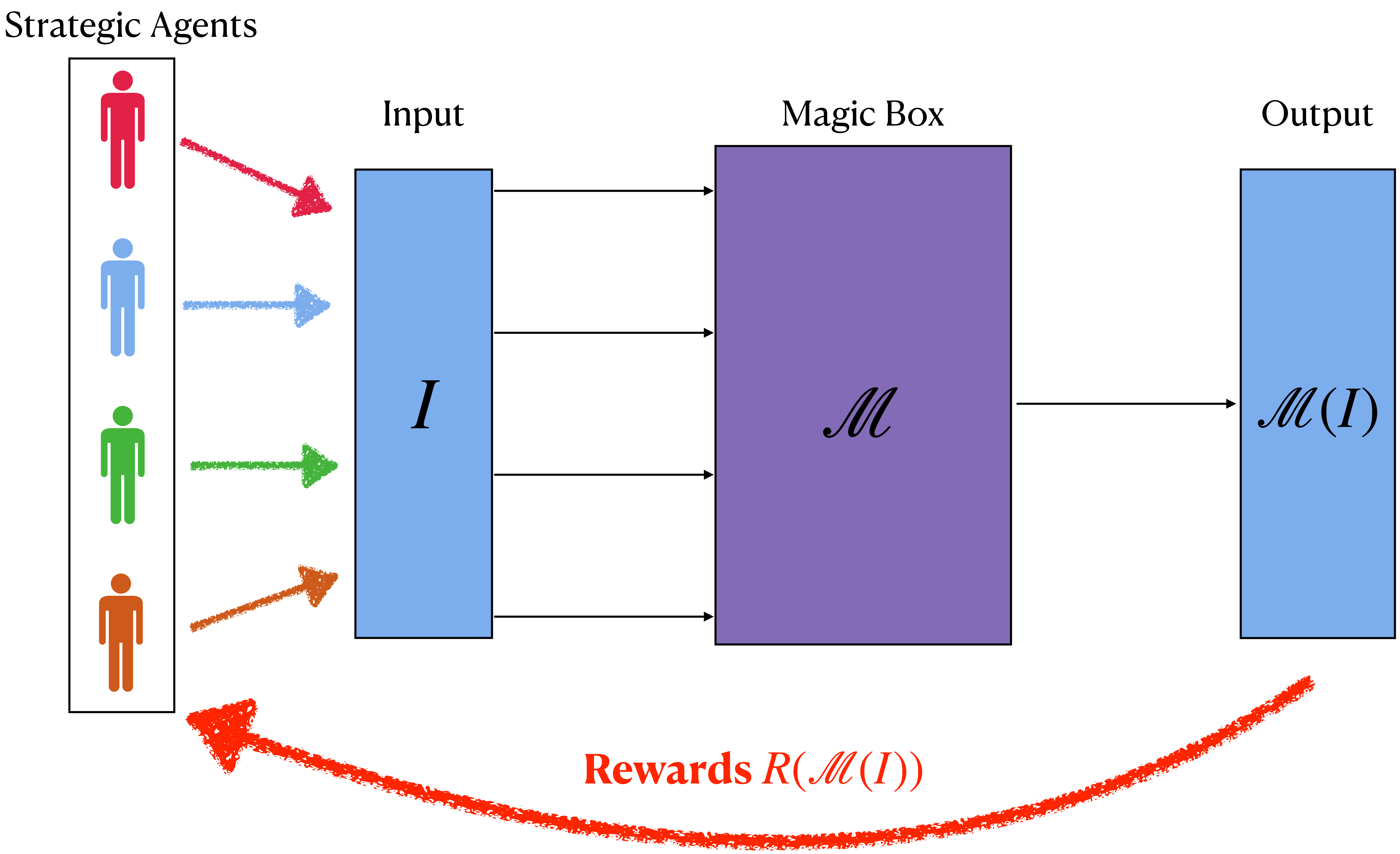
Talk Outline

1. Mechanism Design Fundamentals.
2. Bilateral Trade under this Lens.
3. Value Assumptions.
4. Bilateral Trade with Interdependent Values.

An algorithmic framework



Mechanism Design Task

- **Same setup** as in algorithm design (Input, Output, Objective).
- **Additional Constraints**:
  - **Incentive Compatibility (IC)**: It is in the best interest of participating agents to report their true information in the mechanism.
  - **Individual Rationality (IR)**: Participating in the mechanism can only be beneficial for an agent.
- How do we (usually) enforce these constraints? **Payments**.

Mechanism Design Examples

Auctions

Public Projects

Matching Doctors to Hospitals

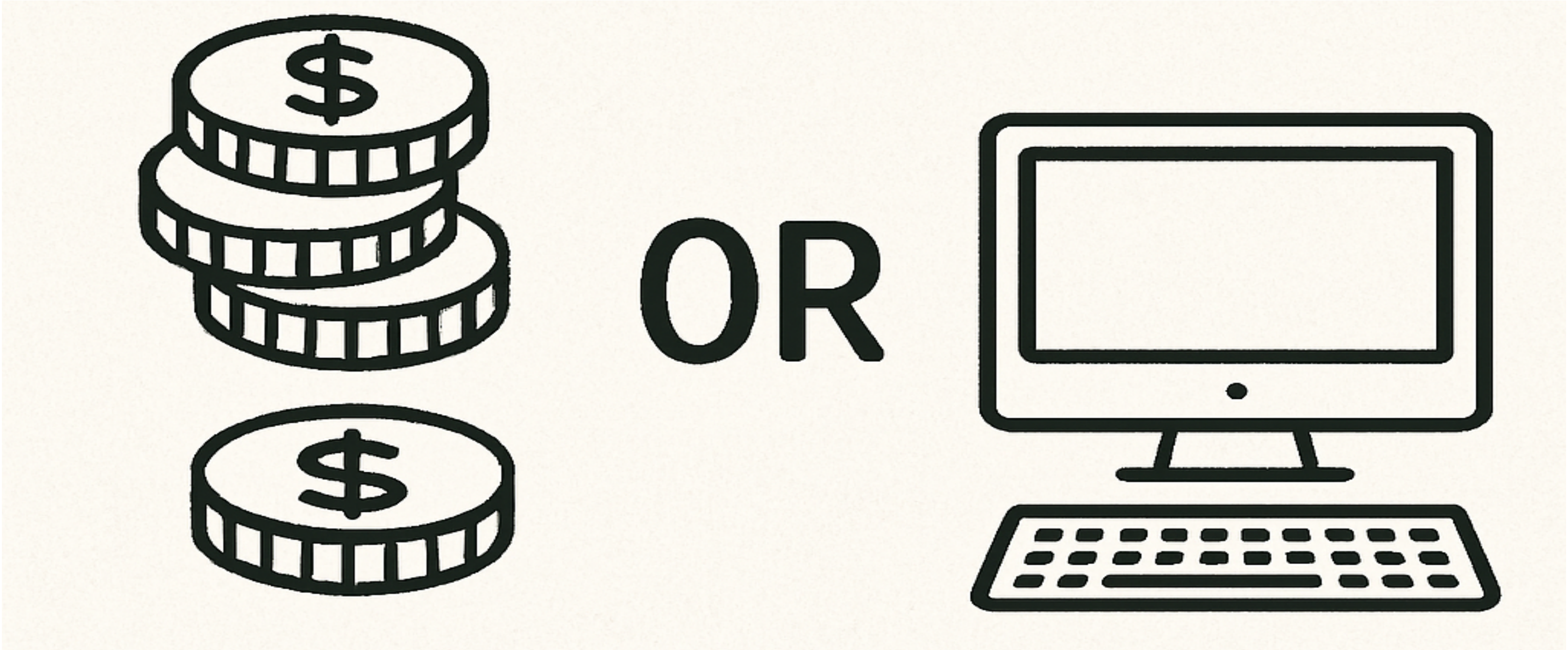
Voting

Contract Design

Mechanism Design under Economics & Computer Science

- **Economists** started studying mechanism design in the 1960's (Hurwicz, Maskin, Myerson).
- **Lens of study**: Existence and characterization of optimal mechanisms.

- **Computer Scientists** picked up (algorithmic) mechanism design in the 1990's (Nisan, Ronen, Roughgarden, Tardos, Papadimitriou).
- **Lens of study**: Efficient computation in mechanism design, approximately optimal mechanisms



Bilateral trade

Setting: A seller with one item and a potential buyer. The seller values the item v_s and the buyer values it v_b, and these values are **private information**, drawn from **publicly** known distributions.

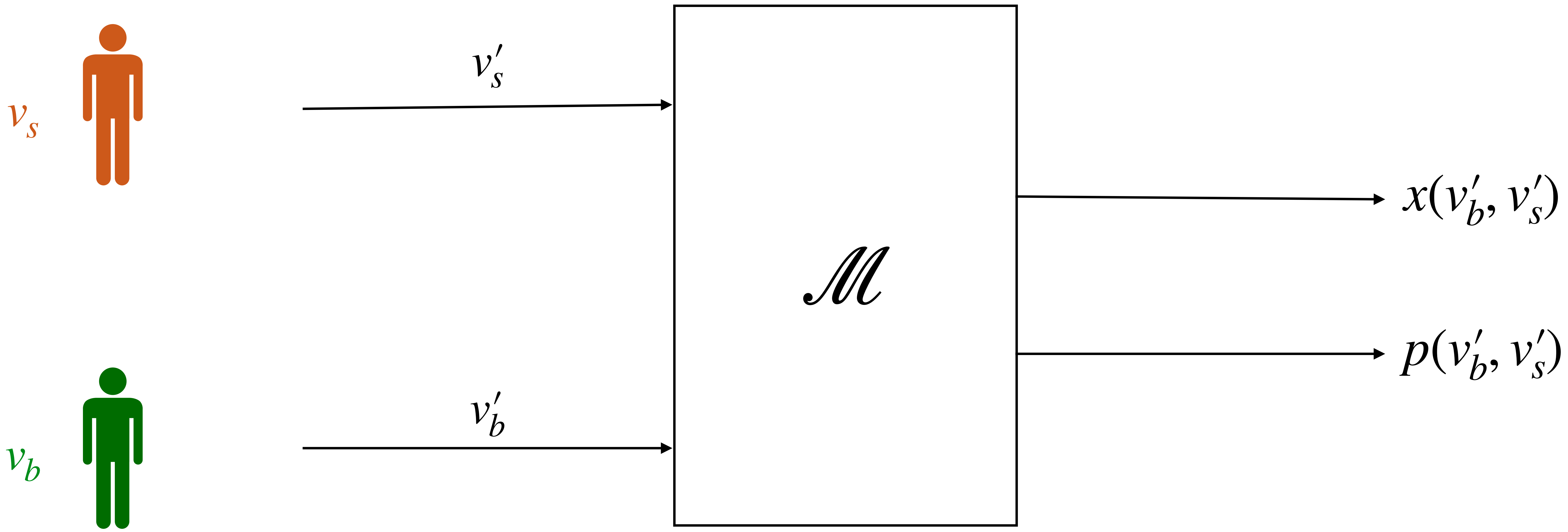
Designer goal: Decide if the trade should happen and under what payment scheme.

Natural objective: Trade whenever the buyer values the item more than the seller ($v_b > v_s$).

Mechanism Design Task

A (direct) **mechanism** $\mathscr{M}$ consists of two functions $\mathscr{M} = (x, p)$, takes **reported** values v'_b and v'_s as input and outputs:

1. The **probability** of trade $x(v'_b, v'_s)$.
2. The **price** that the buyer pays to the seller $p(v'_b, v'_s)$.



Agents & Constraints

Utilities under mechanism $\mathscr{M}$ with reports (v_b', v_s'):

Seller utility: $p(v_s', v_b') - v_s \cdot x(v_s', v_b')$ Buyer Utility: $v_b \cdot x(v_s', v_b') - p(v_s', v_b')$

Desired Constraints:

payment – value · probability of trade

- **Individual Rationality** (IR) -----> Non-negative utility from participating in the mechanism.
- **Incentive Compatibility** (IC) --> No incentive to misreport my information to the mechanism.
- **Budget Balance** (BB) ------------> The designer does not subsidize the trade.

Objectives

A mechanism's **performance** in bilateral trade is commonly measured in:

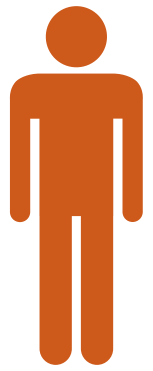
1. Social Welfare: the **value** (welfare) of the agent that is allocated the item.

   An optimal mechanism achieves $SW = \mathbb{E}\left[max(v_b, v_s)\right]$.

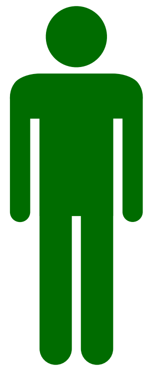
2. Gains from Trade: the **welfare increase** due to the trade (if it happens).

   An optimal mechanism achieves $GFT = \mathbb{E}\left[max(v_b - v_s, 0)\right]$.

A Simple Example



$$v_s = \begin{cases} 4, & \text{w.p. } \frac{1}{2}, \\ 0, & \text{otherwise}. \end{cases}$$



$$v_b = 2.$$

Distribution

<u>At Optimality</u>:

- <u>Expected Welfare</u>:

$$SW = \mathbb{E}\left[max(v_b, v_s)\right] = 3.$$

- <u>Expected GFT:</u>

$$GFT = \mathbb{E}\left[max(v_b - v_s, 0)\right] = 1.$$

Optimal Objectives

<u>Example mechanism</u>:

No **trade** mechanism: $(x, p) = (\mathbf{0}, \mathbf{0})$.

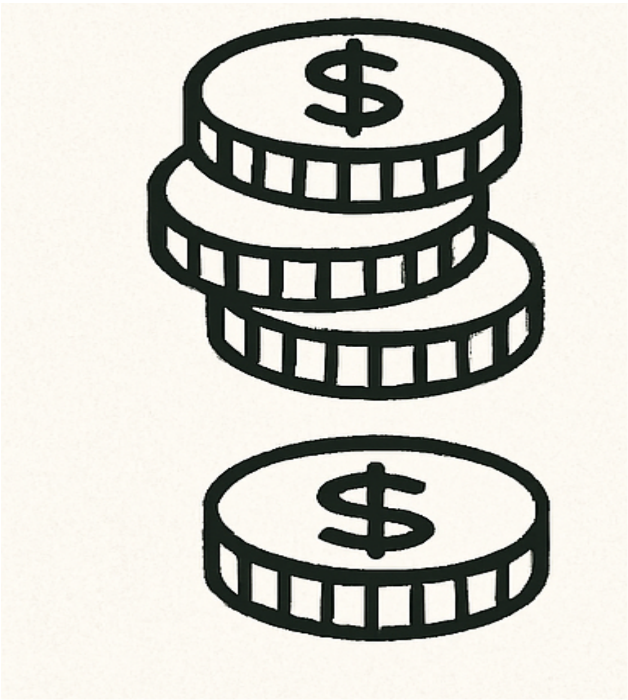
- <u>Expected Welfare</u>:

$$\mathbb{E}[Welfare] = 2.$$

- <u>Expected GFT:</u>

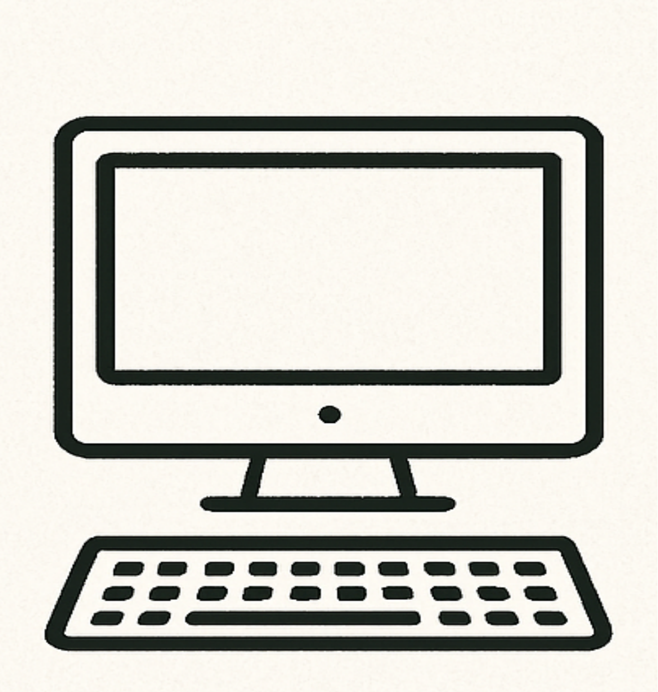
$$\mathbb{E}[Gains\ from\ trade] = 0.$$

No trade mechanism $\mathcal{M}$



Economists: Optimality is unattainable

(Informal) **Theorem [Myerson-Satterthwaite 83]:** There exists no mechanism that simultaneously guarantees individual rationality, incentive compatibility, budget balance, and maximizes Social Welfare.



Computer Scientists: Approximation Thrives

Independent Values:

- Welfare:
  - Multiple works with posted price mechanisms [Blumrosen and Dobzinski, 2014, 2021, **Cai and Wu, 2023**, Colini-Baldeschi et al., 2016, **Kang et al., 2022**, **Liu et al., 2023**].
  - State of the Art (**blue**) is a **1.38** approximation, Lower bound (**red**) is **1.354**.
- Gains From Trade:
  - Again multiple works with posted price mechanisms [**Babaioff et al., 2021**, 2020, Blumrosen and Dobzinski, 2014, Brustle et al., 2017, Cai et al., 2021, Deng et al., 2022, **Fei, 2022**, McAfee, 2008]
  - State of the Art (**blue**) achieves a **3.15** approximation, Lower bound (**red**) is **1.358**.

Correlated Values - Welfare:

- Only one work [Dobzinski and Shaulker, 2024], that proves that a posted price mechanism achieves a **tight 1.582** approximation.

Interdependent Bilateral Trade: Information vs Approximation

Joint work with Shahar Dobzinski[1], Alon Eden[2], Kira Goldner[3], Ariel Shaulker[1]

[1] Weizmann Institute of Science, [2] Hebrew University of Jerusalem, [3] Boston University

A more realistic example?

An art connoisseur is considering selling their marble sculpture to a civil engineer.

Trade ?

Knows Signal s:
The "artistic" value of the sculpture.

Knows Signal b:
The "material" value of the sculpture.

What if they both care about the other agent's information?

The Interdependent Values Model

The seller has a **private** signal s and the buyer has a **private** signal b. The signals are drawn from **publicly** known distributions.

Their values for the item are **public functions** of the signals, that is the seller's value is $v_s(b , s)$ and the buyer's value is $v_b(b , s)$.

Why should you care about this model?

- Thodoris said so and he seems like he knows his stuff?

1. The model **naturally generalizes** the independent and correlated values model.
2. Milgrom & Weber were awarded the **Nobel Prize** in **Economics** in 2020 for introducing and working in the interdependent values model [1982].

3. You might want to **train** AI agents/neural nets/models to participate in bilateral trade (or other mechanisms).

Amount of Information vs Approximation

- **<u>Information structures</u>** comprise of **additively separable valuations**, with signals b, s drawn **independently** from $U[0,1]$:

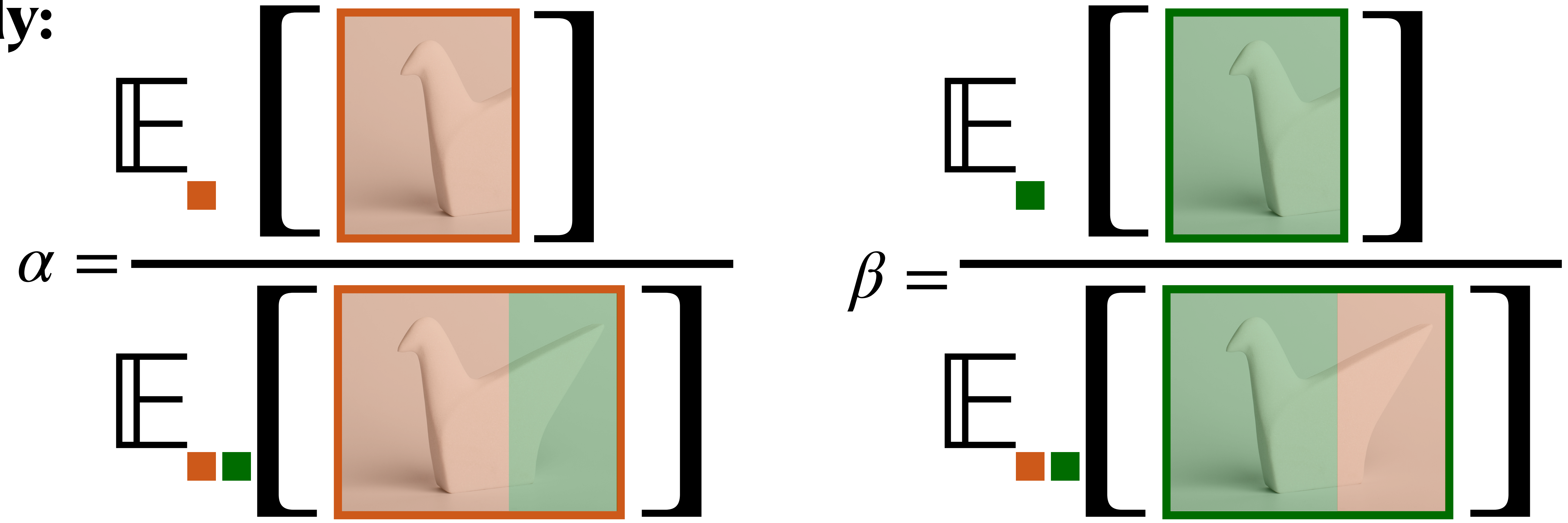
$$v_s(b, s) = f_s(b) + g_s(s), \qquad v_b(b, s) = f_b(b) + g_b(s)\ ,$$

where, $f(\cdot)$, $g(\cdot)$ are non-negative, increasing functions.

- On an information structure, we quantify the **influence** that a player's **private signal has on their own valuation** with parameters α for the seller and β for the buyer.

Amount of Information vs Approximation

- **Defining (α,β) pictorially:**



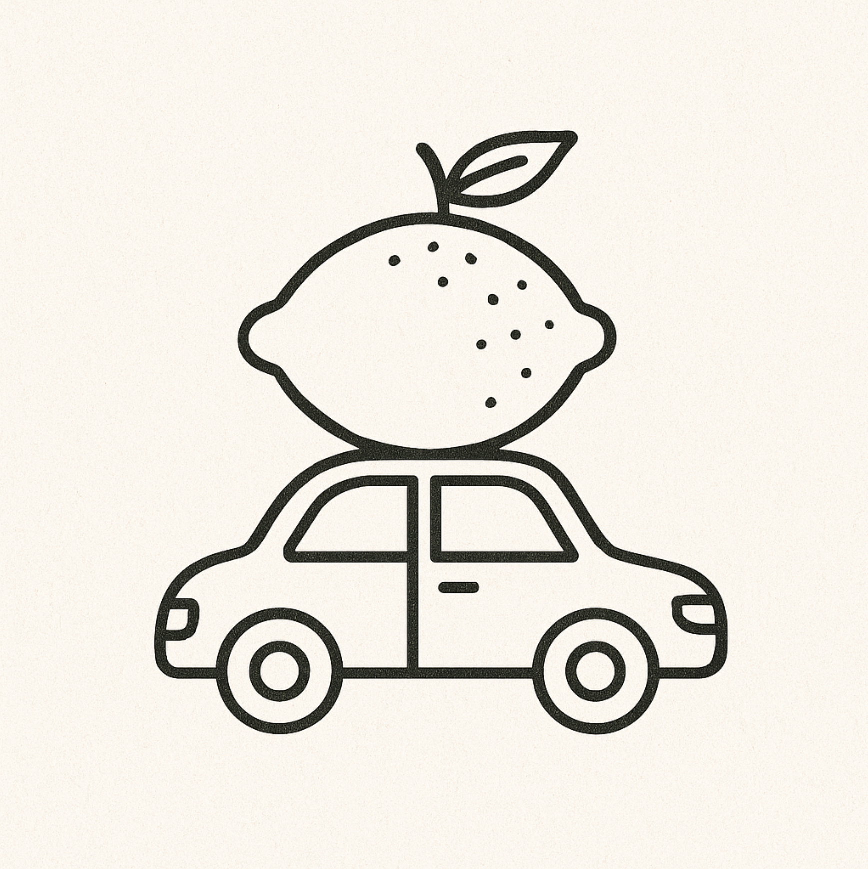
- **Uninformed** seller corresponds to $\alpha = 0$. **Fully informed** seller corresponds to $\alpha = 1$ (same for buyer).

- Formally, we denote the seller α**-informed** and the buyer β**-informed** with:

$$\alpha = \frac{\mathbb{E}_s[v_s(0,s)]}{\mathbb{E}_{s,b}[v_s(b,s)]}, \qquad \beta = \frac{\mathbb{E}_b[v_b(b,0)]}{\mathbb{E}_{b,s}[v_b(b,s)]}.$$

Information Asymmetry - The Market for Lemons [Akerlof' 70]

Consider we want to design bilateral trade mechanisms for a **used car trade**. Assume that cars in the market are **evenly** divided into:



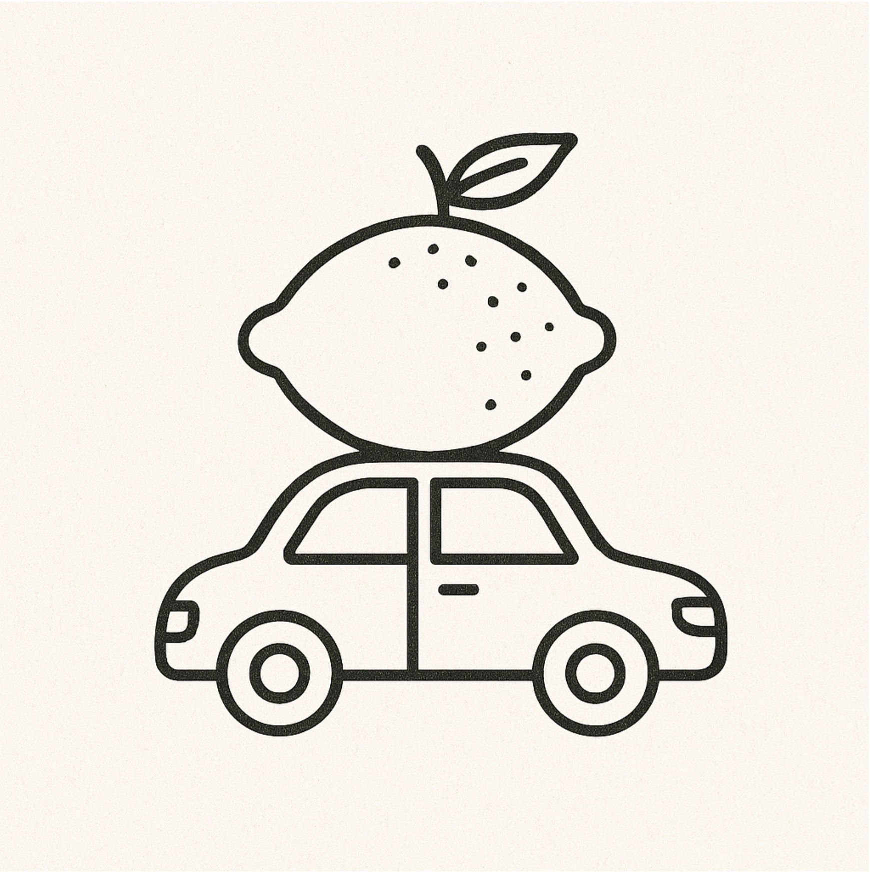
- ***Peaches***: Cars in excellent condition - valued at $10000.
- *Lemons:* Cars in terrible condition - valued at $0.

- The seller and the buyer share the same value function for the cars. However:
  - The seller has complete information of whether their car is a ***peach*** or a *lemon*.
  - The buyer has no information whatsoever.
- This instance corresponds to a (0,1)-information structure (that is the seller is uninformed and the buyer is fully-informed).

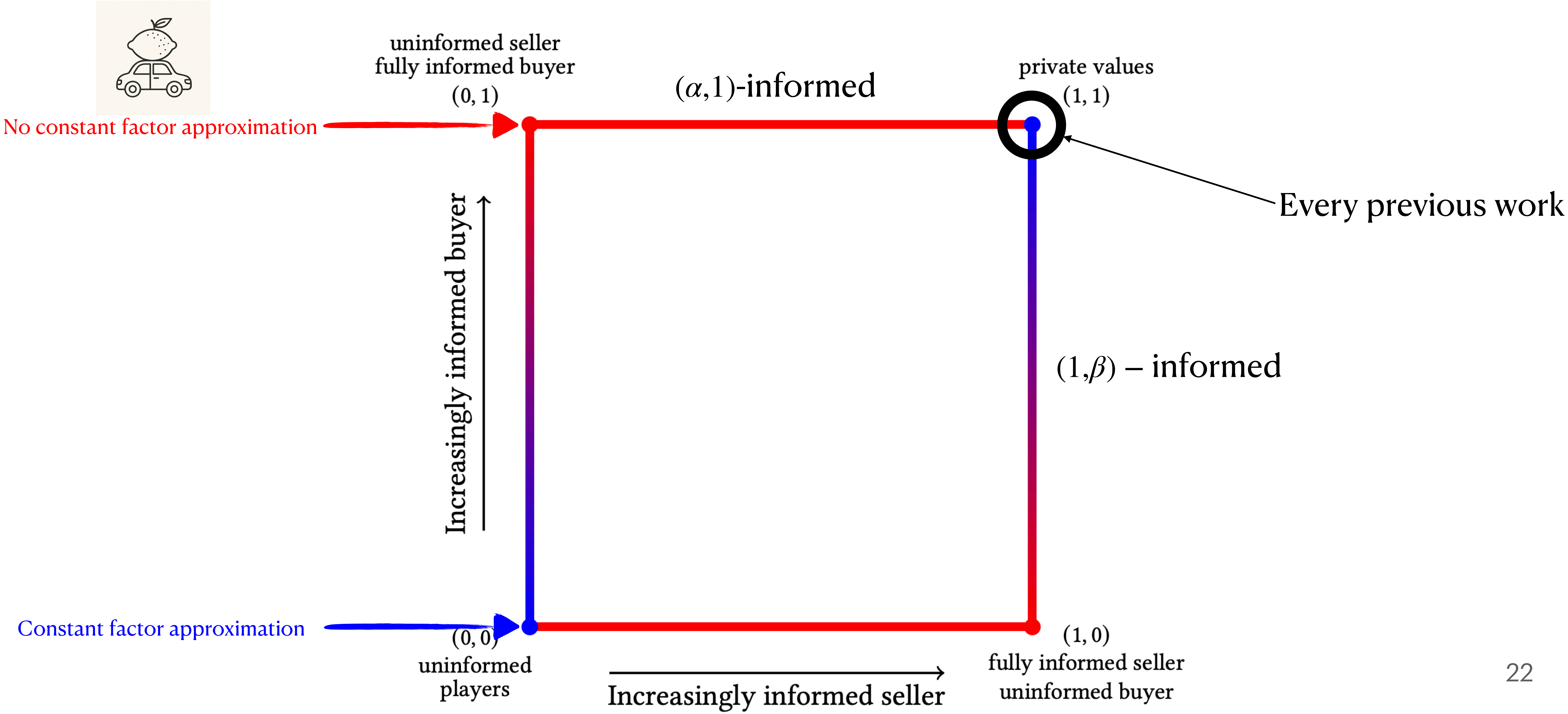
Information Asymmetry - The Market for Lemons [Akerlof' 70]

Consider a simple mechanism that posts a price of $3000:

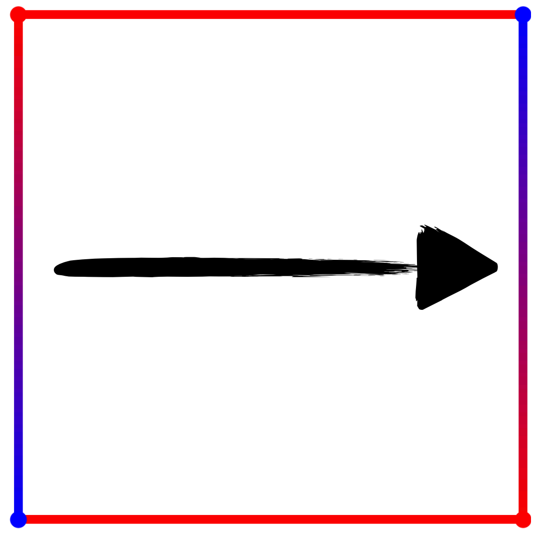
- When should the seller accept this price?
  - Accept the trade if your car is a *lemon* (worth $0).
  - Reject the trade if your car is a ***peach*** (worth $10k).
- When should the buyer accept this price?
  - **Always reject**. The buyer should be conditioning on the seller accepting the trade.
- What is the expected welfare of this mechanism? What is the the optimal welfare?



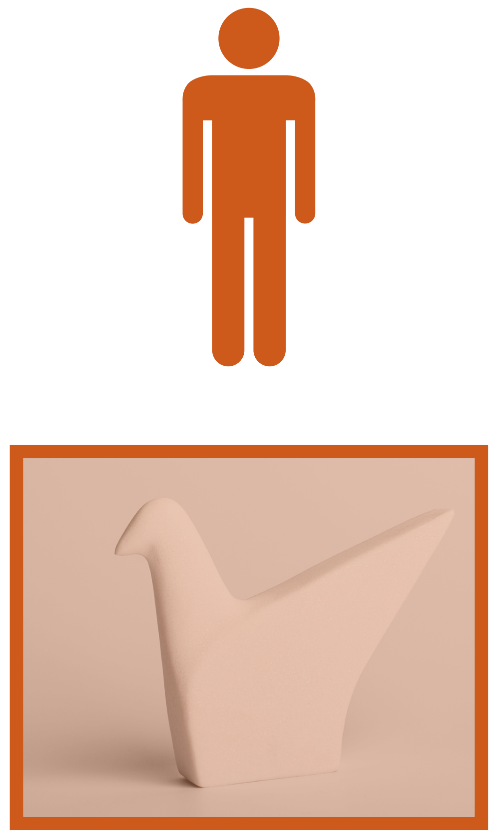
Overview of results for (α,β) - information structures on the square

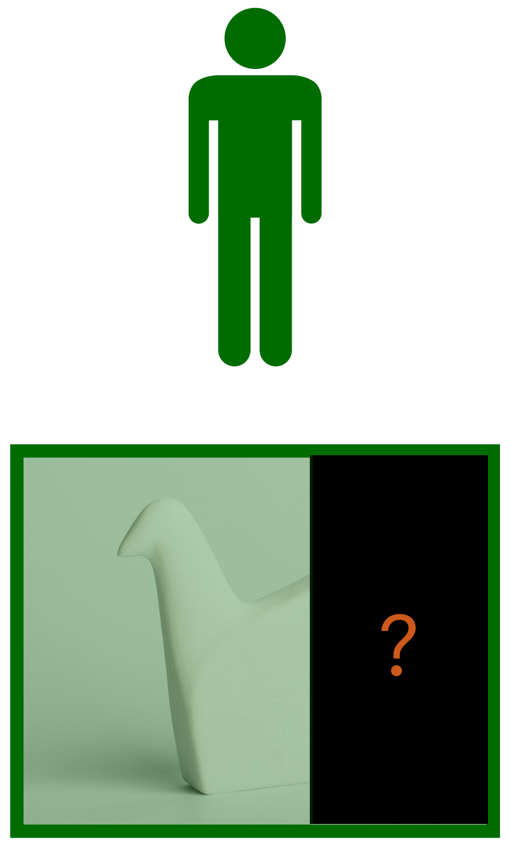


A Fully Informed Seller: $(1, \beta)$ Edge



(Informal) Theorem 1: Let $\mathscr{M}$ be a posted price mechanism for the (private) independent values case with an approximation ratio of γ. Consider an information structure with $\beta > 0$ and a fully informed seller ($\alpha = 1$). Then there **exists** a BIC **mechanism** $\mathscr{M}'$ with an **approximation** ratio of $\dfrac{2\gamma}{\beta}$.





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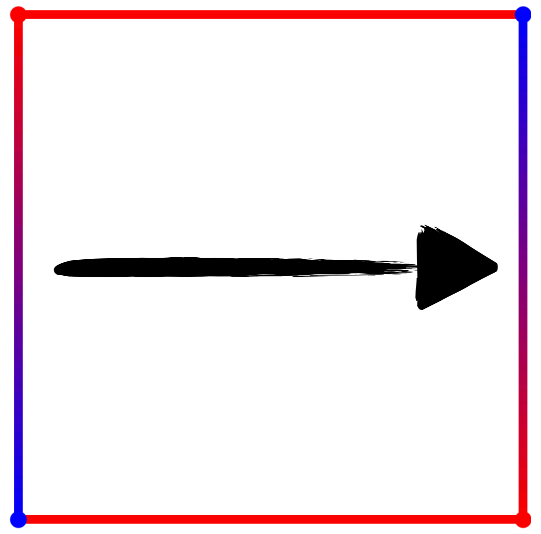
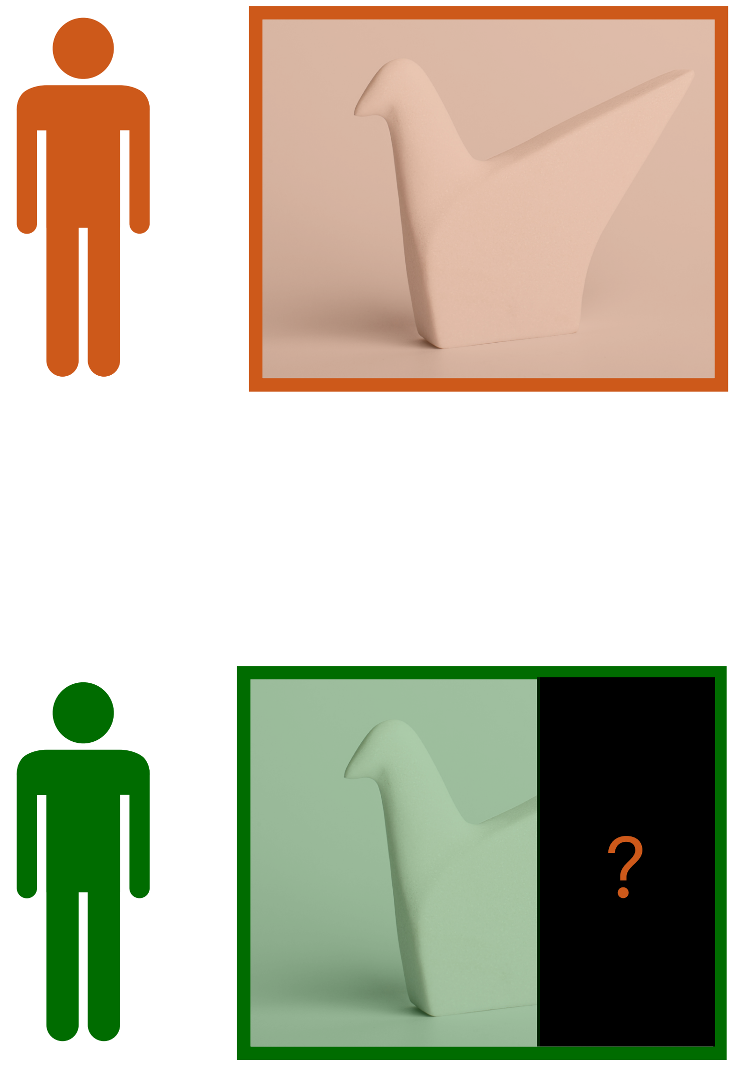
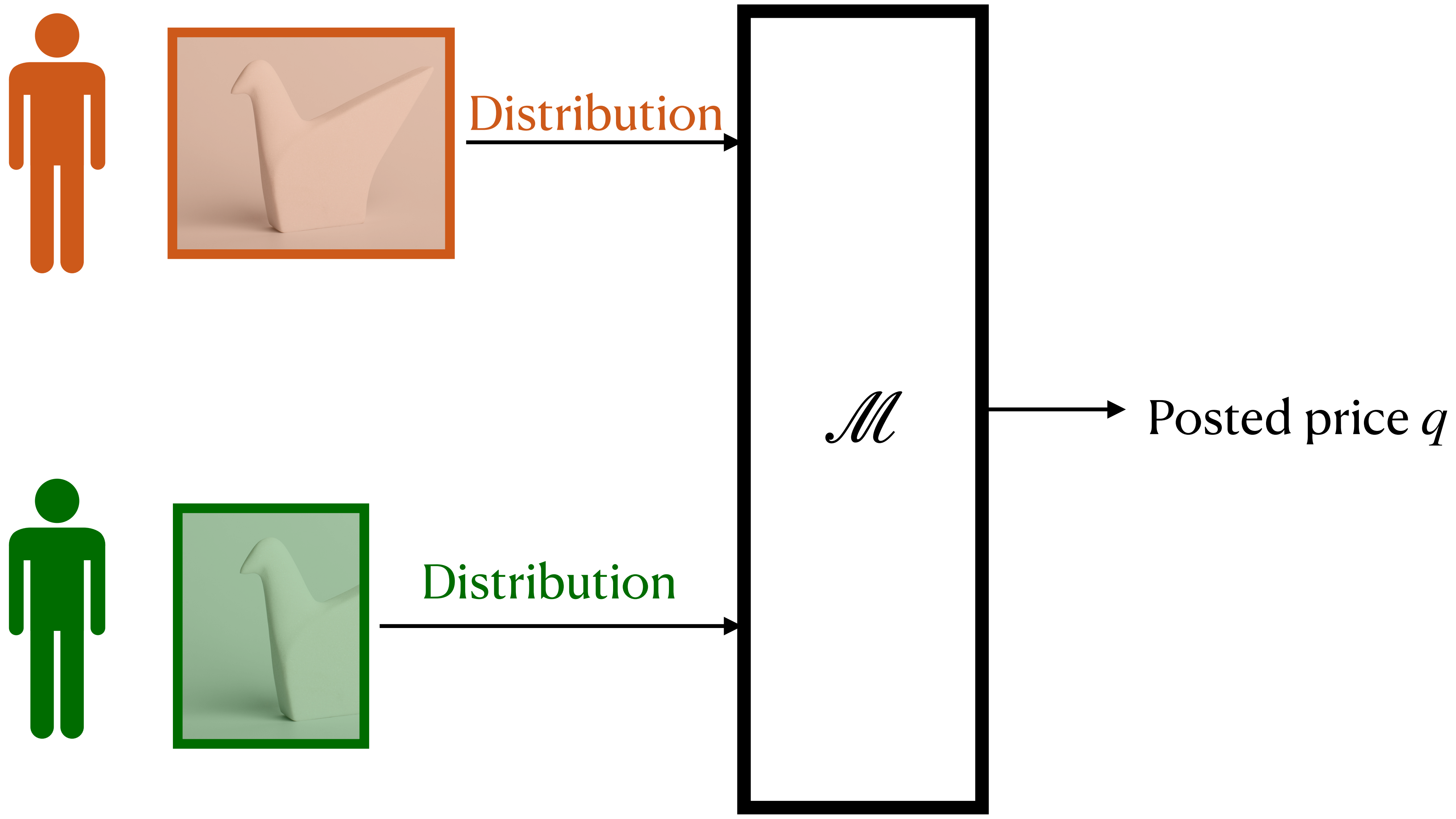


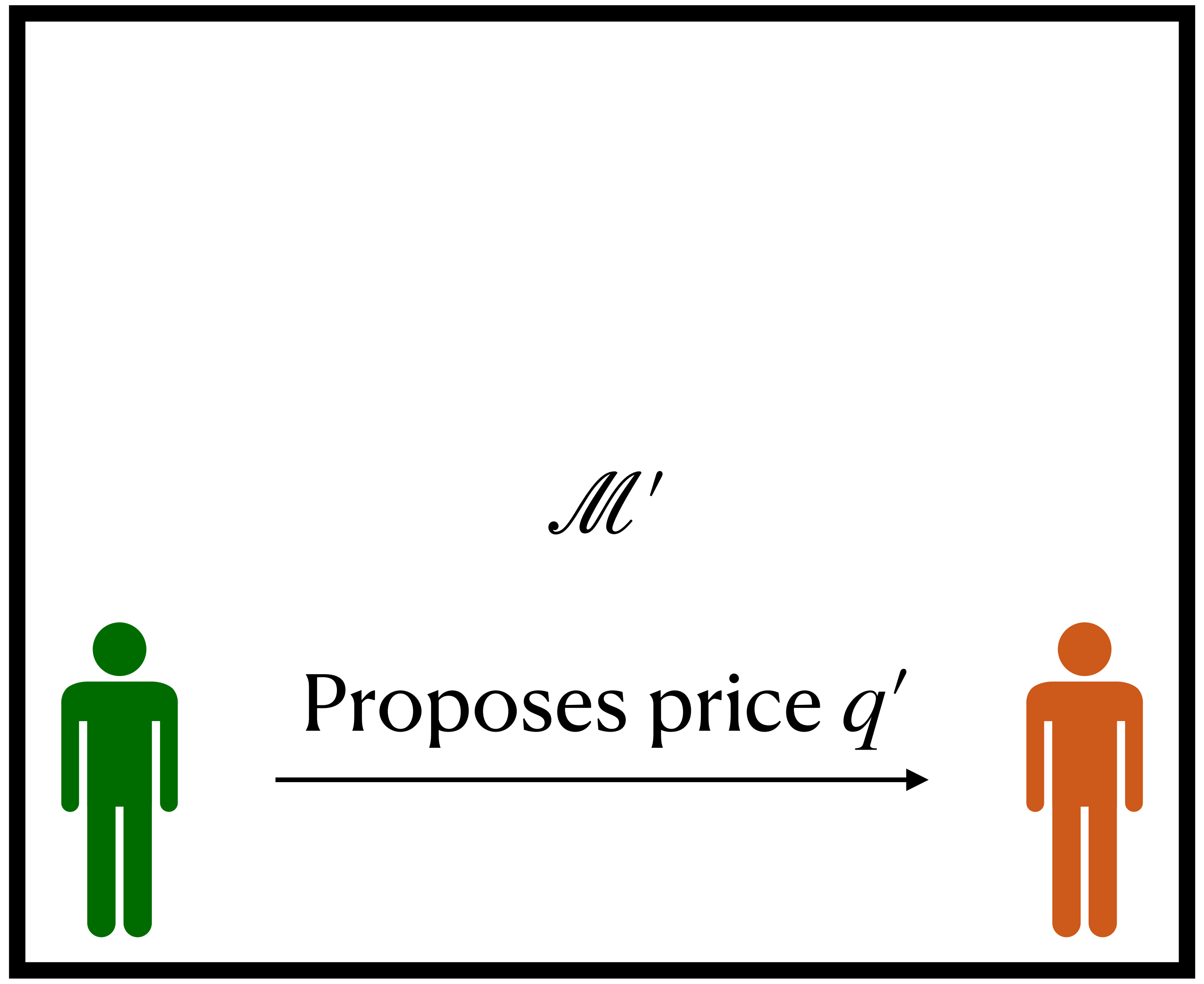
Proof Sketch - Compare Two Posted Price Mechanisms:



True Instance

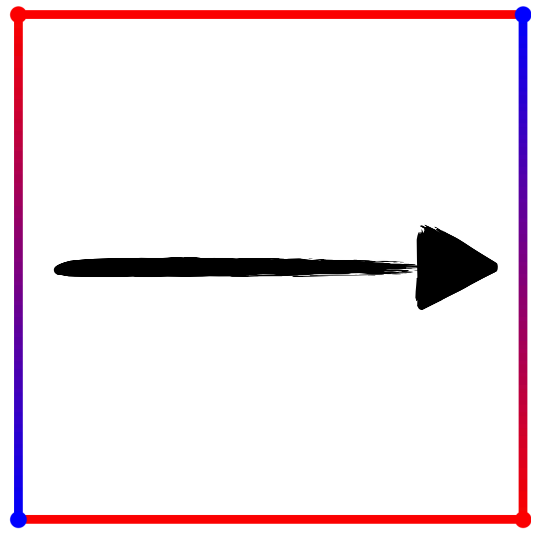


Independent Values Instance

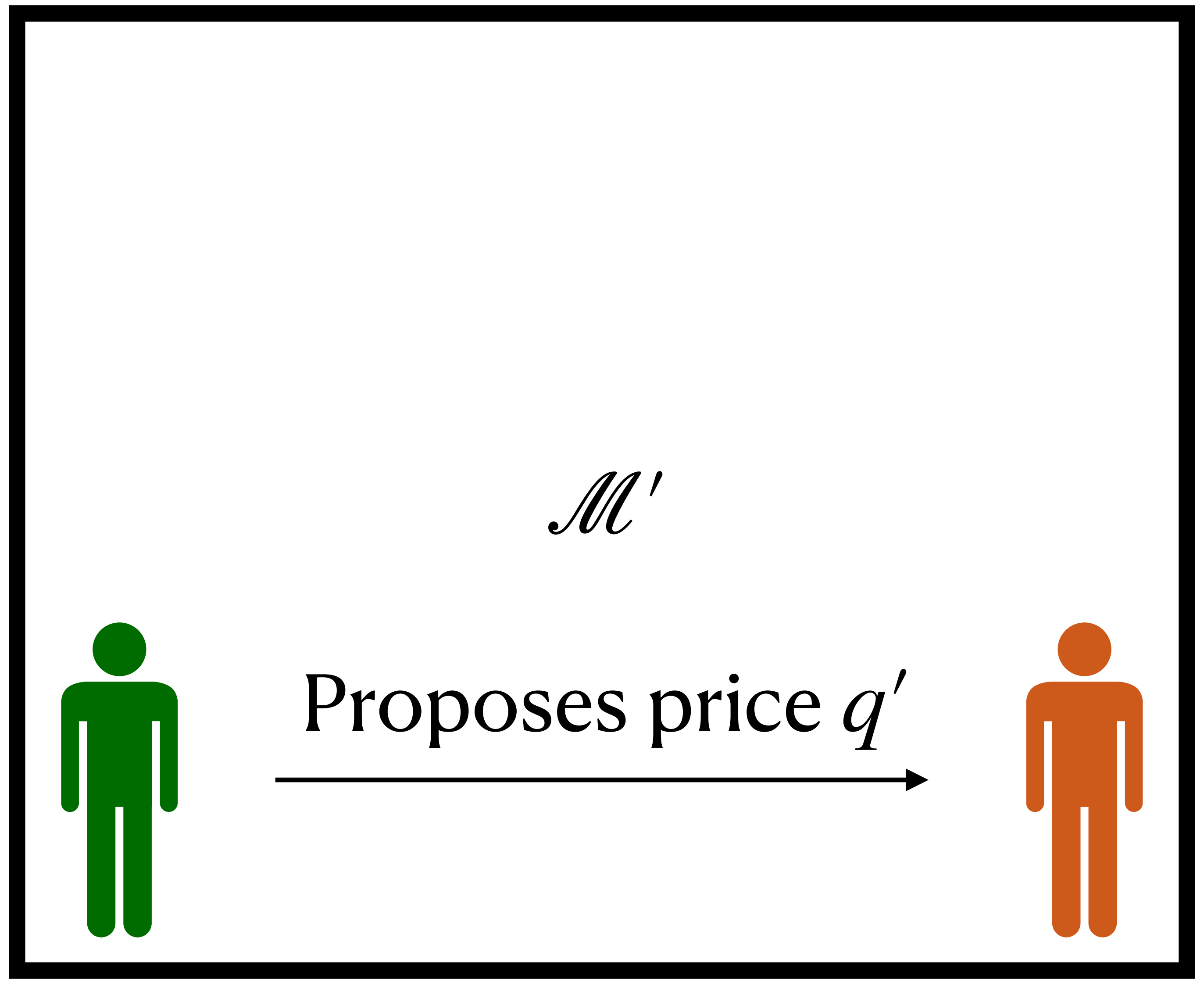


Interdependent Mechanism

A Fully Informed Seller: (1, β) Edge



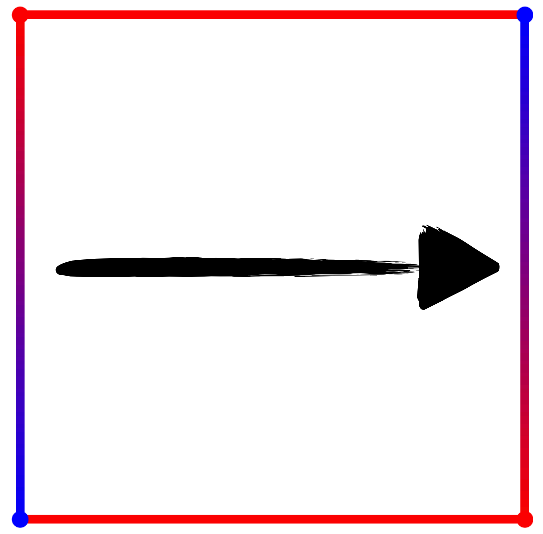
Proof Sketch - Compare Two Posted Price Mechanisms:



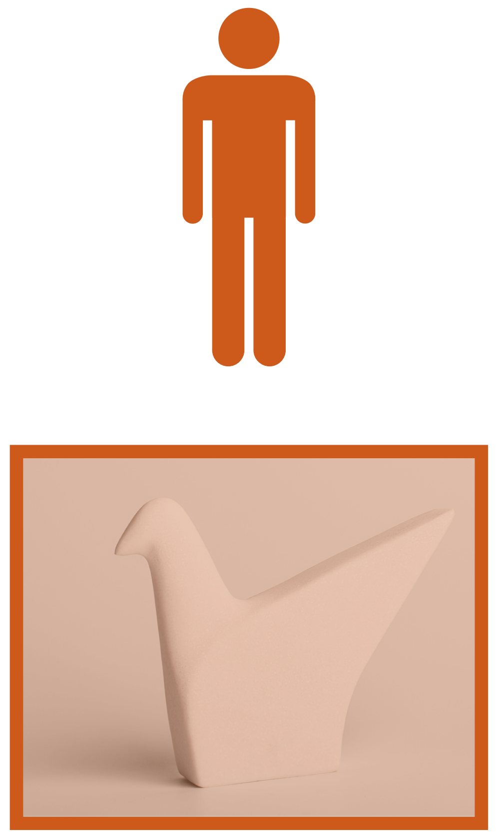
Investigating $\mathscr{M}'$:

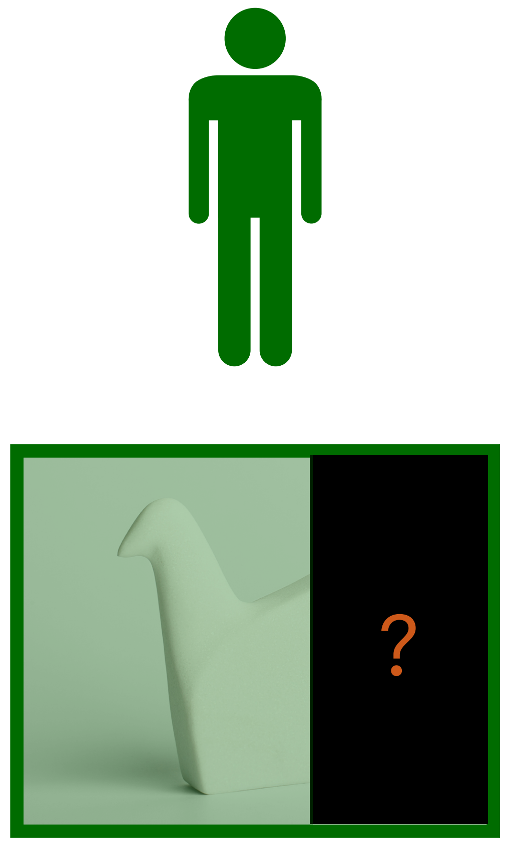
- The buyer proposes the price q' so $\mathscr{M}'$ satisfies **BIC** and **interim IR** (for the buyer).
- The seller is fully informed and responds to the proposed price optimally (so seller **BIC** and **interim IR** are also guaranteed).
- The **proposed price** q' can only be **higher** than price q (the price posted by the independent values mechanism $\mathscr{M}$).
- This implies the Welfare of $\mathscr{M}'$ is at **least as large** as the Welfare of $\mathscr{M}$.

A Fully Informed Seller: $(1, \beta)$ Edge



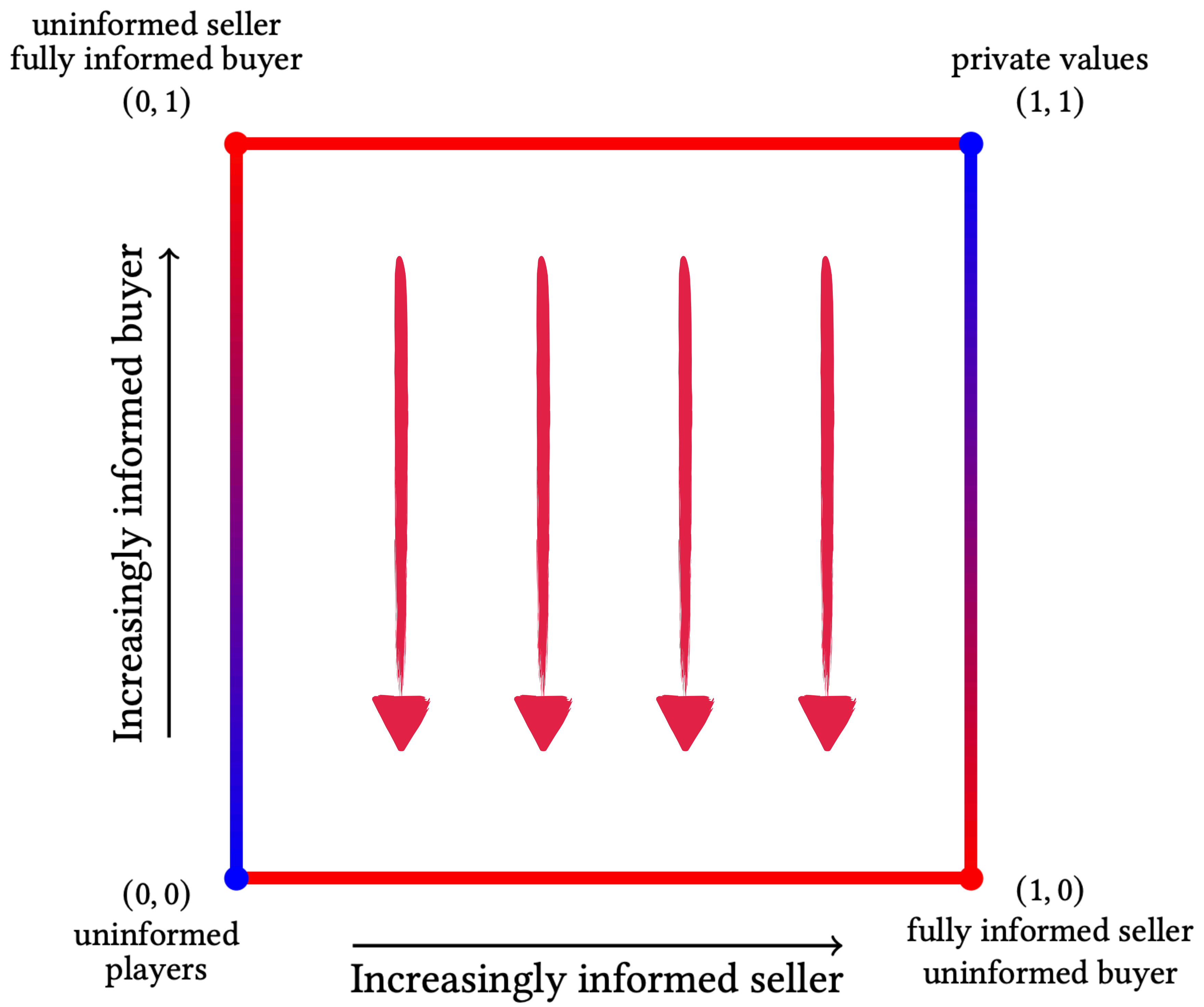
(**Formal**) **Theorem 2:** For every $\beta \in (0,1)$, there exists an information structure where the seller is fully informed and the buyer is β-informed, and **no** BIC and interim IR **mechanism can provide an approximation ratio** better than $\frac{2}{3\beta}$.





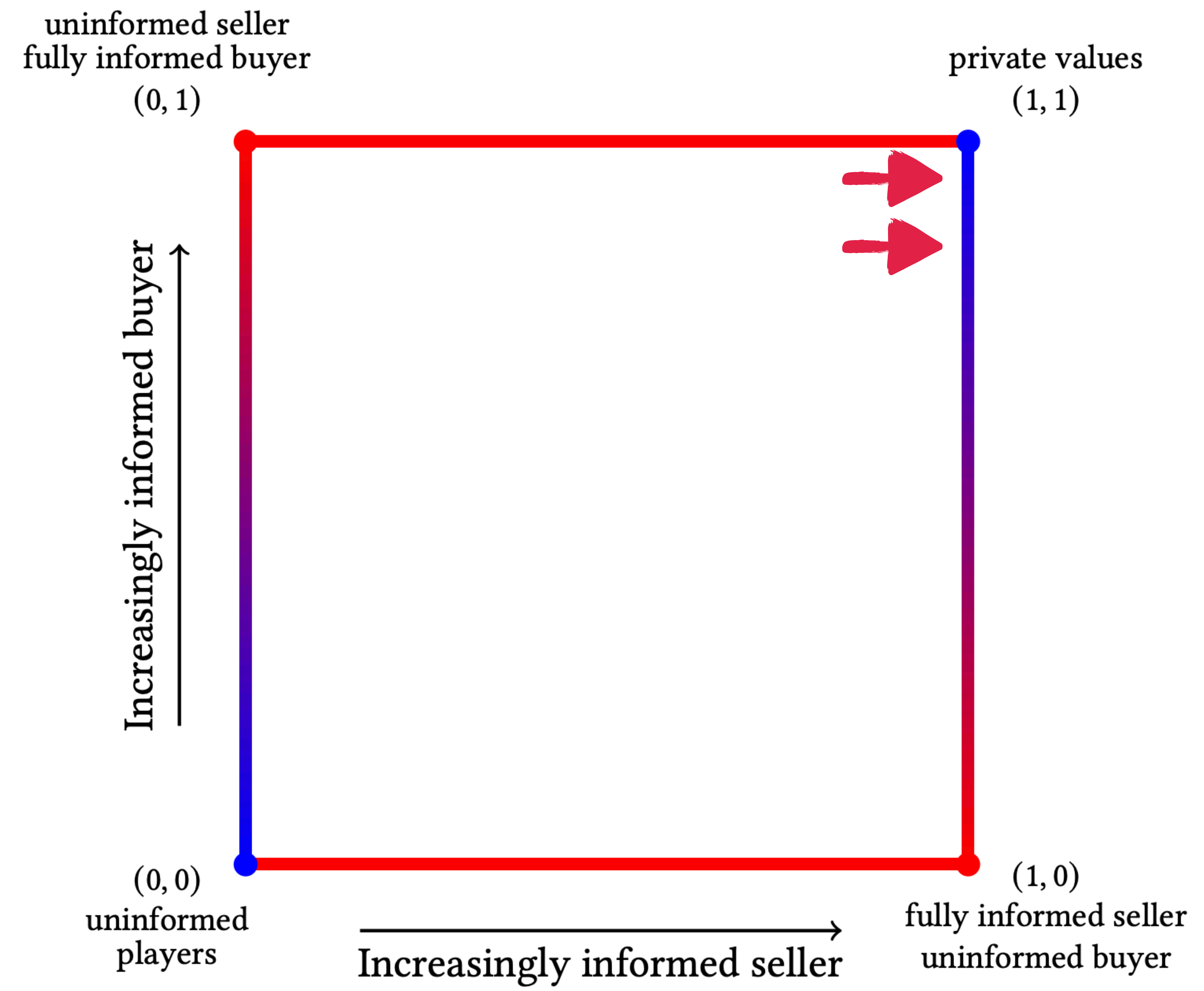
Interior of the square, overview of results

(**Formal**) **Proposition 7:** For every $\alpha > 0$ and $\beta < 1$, there exists an (α, β)-information structure where no BIC and interim IR mechanism can provide an approximation ratio better than $\frac{1}{2\beta}$.



Interior of the square, overview of results

(Formal) Proposition 8: For every $\alpha \in (0.9,1)$ and $\beta \in [1-(1-\alpha)^3, 1)$, there exists an (α, β)-information structure where no BIC and interim IR mechanism can provide an approximation ratio better than $\dfrac{0.15}{1-\alpha}$.



Families of Information Structures - Polynomials

- Assume that the valuations of the buyer and the seller are **polynomials** of the signals, of **maximum degree** k:

$$v_s(b,s) = \sum_{i=1}^{k} c_i \cdot s^i + \sum_{i=1}^{k} d_i \cdot b^i + c_s\,,$$

and

$$v_b(b,s) = \sum_{i=1}^{k} a_i \cdot b^i + \sum_{i=1}^{k} b_i \cdot s^i + c_b\,.$$

- The signals b, s are **independently** drawn from $U[0,1]$.

Polynomials - Results

(Formal) **Theorem 9:** Suppose that v_b, v_s are polynomials of maximum degree k, and that the signals are independently drawn from a uniform distribution over $[0,1]$. Then, there **exists** a BIC and interim IR **mechanism that guarantees an approximation** ratio of $O(k^2)$. In particular, when v_b, v_s are linear functions, the approximation ratio is constant.

(Formal) **Theorem 9:** For every $k \in \mathbb{N}$, there exist polynomials v_b, v_s of degree k such that **no** BIC and interim IR mechanism **can achieve an approximation ratio better** than k.

Polynomials - Approximate mechanism

Mechanism $\mathscr{M}$:

1. If $\mathbb{E}[v_s] \geq \dfrac{\mathbb{E}[v_b]}{(k+1)^2}$: Do not trade the item.

2. If $\mathbb{E}[v_s] < \dfrac{\mathbb{E}[v_b]}{(k+1)^2}$: Post a price of $q = \dfrac{\mathbb{E}[v_b]}{k+1}$ (the seller always agrees, the buyer might always agree, or might sometimes agree).

Future Directions

1. **Tightly** characterize what is possible in the interior of the **square**.
2. Consider a **different definition** of **informedness** in information structures.
3. Investigate what is possible for the **GFT** objective.
4. Study **other families** of information structures.
5. Move beyond bilateral trade to **two-sided markets** (multiple buyers and/or sellers).

Summary

- Introduced the field of **mechanism design** and the problem of **bilateral trade**.
- Discussed **value assumptions** in mechanism design.
- Provided mechanisms and impossibilities for **(α,β)-information structures**.
- Got a little bit confused.

Thank you !